VOLUME 1

APPLIED PHYSICS

Electricity and Magnetism

Electric Charge • Electric Fields • Gauss's Law

Electric Potential • Current and Resistance

Magnetic Fields • Electromagnetic Induction

Maxwell's Equations

Dr. Irfan Toqeer

PhD Physics, MSc Computer Science

First Edition • 2025

APPLIED PHYSICS

Volume 1

Electricity and Magnetism

Dr. Irfan Toqeer

Assistant Professor
Department of Physics
Riphah International University, Faisalabad

First Edition

2025

Applied Physics - Volume 1: Electricity and Magnetism

First Edition, 2025

Author:
Dr. Irfan Toqeer
Assistant Professor, Department of Physics
Riphah International University, Faisalabad

Disclaimer:
The information in this book is provided for educational purposes. While every effort has been made to ensure accuracy, the author and publisher assume no responsibility for errors or omissions.

For course information and updates, contact:
Department of Physics, Riphah International University, Faisalabad-Pakistan
Email: irfan.toqeer@riphahfsd.edu.pk

Dedicated to

All Students of Physics

who seek to understand the fundamental laws governing our universe

and to

My Family and Teachers

for their endless support and inspiration

Preface

Physics is the foundation of all natural sciences, and electricity and magnetism represent one of its most profound and practical branches. From the smallest electronic devices in our pockets to the vast power grids that illuminate our cities, electromagnetic phenomena shape virtually every aspect of modern civilization.

This textbook, *Applied Physics - Volume 1: Electricity and Magnetism*, has been specifically designed for undergraduate students at Riphah International University, Faisalabad, and other institutions offering applied physics courses. The primary goal is to provide a clear, comprehensive, and pedagogically sound introduction to electromagnetic theory and its applications.

Philosophy and Approach

The approach taken in this volume emphasizes three key principles:

1. Conceptual Understanding: Before diving into mathematical formalism, each chapter begins with the physical concepts and intuitive explanations. I believe that true mastery comes from understanding *why* things happen, not just *how* to calculate them.

2. Mathematical Rigor: Once concepts are established, detailed mathematical derivations are presented step-by-step. No steps are skipped, and each transformation is explained. This is particularly important for students who may not have strong mathematical backgrounds.

3. Practical Applications: Throughout the text, real-world applications and modern technologies are discussed. Students learn not just abstract theory, but how these principles power our technological world.

Organization of the Book

This volume is organized into logical, progressive chapters:

- **Chapters 1-2** establish the foundation of electrostatics, including electric charge, force, and fields.
- **Chapter 3** introduces Gauss's Law, one of Maxwell's equations, and its applications.
- **Chapter 4** covers electric potential and energy, connecting to earlier work on fields.
- **Chapter 5** transitions to current electricity, resistance, and Ohm's law.
- **Chapters 6-7** explore magnetism, magnetic forces, and the generation of magnetic fields.

- **Chapter 8** presents electromagnetic induction and Faraday's law.
- The volume culminates with Maxwell's equations, unifying electricity and magnetism.

Pedagogical Features

To facilitate learning, this book includes several features:

- **Worked Examples:** Over 50 detailed examples with complete solutions demonstrate problem-solving techniques.
- **Practice Problems:** Each chapter contains numerous problems of varying difficulty.
- **Key Point Boxes:** Important concepts are highlighted for quick reference.
- **Visual Aids:** More than 100 diagrams, graphs, and illustrations clarify complex concepts.
- **Summary Sections:** Each chapter ends with a comprehensive summary and list of key equations.

Who Should Use This Book

This textbook is intended for:

- Undergraduate students in physics, engineering, and computer science
- Students preparing for competitive examinations
- Self-learners seeking a comprehensive introduction to electromagnetism
- Instructors looking for a clear, well-organized teaching resource

Prerequisites include basic calculus (differentiation and integration) and introductory mechanics. Vector calculus concepts are introduced as needed.

How to Use This Book

For Students:

- Read each chapter carefully, working through derivations with paper and pencil.
- Study the worked examples before attempting practice problems.
- Pay special attention to Key Point and Important Equation boxes.
- Don't skip the conceptual discussions—they build intuition.
- Form study groups to discuss difficult concepts.

For Instructors:

- The book is designed for a one-semester course meeting 3-4 hours per week.
- Each chapter contains more material than can be covered in one lecture, allowing flexibility.
- Additional problems and solutions are available upon request.
- The logical organization allows chapters to be rearranged if desired.

Acknowledgments

This book would not have been possible without the support and contributions of many individuals:

- My students at Riphah International University, whose questions and feedback shaped this book.
- My colleagues in the Department of Physics for their valuable suggestions and encouragement.
- The administration of Riphah International University, Faisalabad, for supporting this project.
- My family for their patience during the many hours spent writing and revising.

A Note on Units and Conventions

Throughout this book, we use the International System of Units (SI). Vector quantities are denoted in bold ($\mathbf{E}$, $\mathbf{B}$) or with arrow notation ($\vec{E}$, $\vec{B}$). Significant figures in examples follow standard physics conventions.

Feedback and Corrections

Despite careful review, some errors may remain. I welcome feedback from readers—students, instructors, and self-learners alike. Please send corrections, suggestions, or questions to the Department of Physics at Riphah International University, Faisalabad.

This book is a living project. Your input will help improve future editions and make this resource more valuable for students worldwide.

Dr. Irfan Toqeer
Department of Physics
Riphah International University, Faisalabad
September 2025

November 6, 2025

Contents

1 Electric Charge and Electric Force **7**

1.1 Introduction to Electric Charge . . . 7
1.2 Conservation of Charge . . . 7
1.2.1 Examples of Charge Conservation . . . 8
1.3 Charge Quantization . . . 8
1.4 Coulomb's Law: The Electric Force . . . 8
1.4.1 Vector Form of Coulomb's Law . . . 9
1.4.2 Properties of Electric Force . . . 9
1.4.3 Superposition Principle . . . 10
1.5 Applications of Electric Force . . . 10
1.5.1 Atomic Structure . . . 10
1.5.2 Electrostatic Precipitators . . . 10
1.5.3 Inkjet Printers . . . 11

2 Electric Fields **13**

2.1 The Concept of Electric Field . . . 13
2.1.1 Physical Interpretation . . . 13
2.2 Electric Field Due to a Point Charge . . . 13
2.2.1 Derivation . . . 13
2.3 Electric Field Lines (Lines of Force) . . . 14
2.3.1 Properties of Field Lines . . . 14
2.4 Electric Field of Continuous Charge Distributions . . . 15
2.4.1 General Approach . . . 15
2.5 Electric Field of a Ring of Charge . . . 15
2.5.1 Derivation . . . 15
2.5.2 Special Cases . . . 16
2.6 Electric Field of a Disk of Charge . . . 16
2.6.1 Derivation . . . 17
2.6.2 Infinite Plane of Charge . . . 18
2.7 Point Charge in an Electric Field . . . 18
2.7.1 Motion in Uniform Electric Field . . . 18
2.8 Electric Dipole in an Electric Field . . . 18
2.8.1 Dipole Moment . . . 19
2.8.2 Torque on a Dipole . . . 19
2.8.3 Potential Energy of a Dipole . . . 20

3 Gauss's Law **21**
3.1 The Flux of a Vector Field . . . 21
3.1.1 Definition of Flux . . . 21
3.2 The Flux of Electric Field . . . 21
3.2.1 Flux Through a Closed Surface . . . 22
3.3 Gauss's Law . . . 22
3.3.1 Derivation for a Point Charge . . . 22
3.3.2 Key Points . . . 23
3.4 Applications of Gauss's Law . . . 23
3.4.1 Strategy for Using Gauss's Law . . . 23
3.5 Spherically Symmetric Charge Distribution . . . 24
3.5.1 Uniformly Charged Sphere . . . 24
3.6 Field of a Charged Isolated Conductor . . . 25
3.6.1 Field Inside a Conductor . . . 25
3.6.2 Field at the Surface . . . 25

4 Electric Potential and Potential Energy **27**
4.1 Electric Potential Energy . . . 27
4.1.1 Work Done by Electric Force . . . 27
4.1.2 Definition of Potential Energy . . . 27
4.1.3 Potential Energy of Two Point Charges . . . 27
4.2 Electric Potential . . . 27
4.2.1 Physical Meaning . . . 28
4.3 Calculating Potential from the Field . . . 28
4.3.1 Uniform Electric Field . . . 28
4.4 Potential Due to Point Charges . . . 29
4.4.1 Single Point Charge . . . 29
4.4.2 Multiple Point Charges . . . 29
4.5 Potential Due to Continuous Charge Distributions . . . 30
4.5.1 Uniformly Charged Ring . . . 30
4.5.2 Uniformly Charged Disk . . . 30
4.6 Potential Due to an Electric Dipole . . . 30
4.7 Equipotential Surfaces . . . 31
4.7.1 Properties . . . 31
4.8 Calculating Field from Potential . . . 32
4.8.1 Gradient . . . 32

5 Electric Current and Resistance **35**
5.1 Electric Current . . . 35
5.1.1 Direction of Current . . . 35
5.1.2 Microscopic View . . . 35
5.2 Current Density . . . 36
5.3 Resistance and Resistivity . . . 36
5.3.1 Resistance . . . 36
5.3.2 Resistivity . . . 36
5.3.3 Conductivity . . . 36
5.4 Ohm's Law . . . 37
5.4.1 Microscopic Form . . . 37

5.4.2 Power Dissipation . . . 37
5.5 The Hall Effect . . . 37
5.5.1 Applications . . . 38

6 Magnetic Forces and Fields 39
6.1 Magnetic Force on a Current . . . 39
6.1.1 Direction: Right-Hand Rule . . . 39
6.2 The Biot-Savart Law . . . 40
6.2.1 Magnetic Field of a Straight Wire . . . 40
6.2.2 Derivation for Straight Wire . . . 41
6.3 Magnetic Field Lines . . . 41
6.4 Force Between Two Parallel Conductors . . . 42
6.5 Ampère's Law . . . 42
6.5.1 Application: Field of a Solenoid . . . 42
6.5.2 Derivation . . . 43
6.6 Toroid . . . 43

7 Electromagnetic Induction 45
7.1 Faraday's Experiments . . . 45
7.2 Faraday's Law of Induction . . . 45
7.2.1 For N Turns . . . 45
7.3 Lenz's Law . . . 46
7.4 Motional EMF . . . 46
7.4.1 Derivation . . . 46
7.4.2 Alternative Derivation Using Faraday's Law . . . 47
7.5 Induced Electric Field . . . 47
7.5.1 Induced Electric Field from Faraday's Law . . . 47
7.6 Maxwell's Equations: The Basic Equations of Electromagnetism . . . 48
7.7 The Displacement Current . . . 48
7.7.1 The Problem . . . 49
7.7.2 Maxwell's Correction . . . 49
7.7.3 Physical Meaning . . . 49
7.8 Induced Magnetic Field . . . 49
7.8.1 Symmetry Between E and B . . . 50

A Problem Set 53
A.1 Electric Force and Field . . . 53
A.2 Gauss's Law . . . 53
A.3 Electric Potential . . . 53
A.4 Current and Resistance . . . 53
A.5 Magnetism . . . 53
A.6 Induction . . . 54

Chapter 1

Electric Charge and Electric Force

1.1 Introduction to Electric Charge

Electric charge is a fundamental property of matter that gives rise to electromagnetic interactions. The concept of charge is central to understanding electricity, magnetism, and their applications in modern technology.

There are two types of electric charges:

- **Positive charge**: Associated with protons
- **Negative charge**: Associated with electrons

Like charges repel each other, while unlike charges attract. This fundamental principle governs all electromagnetic phenomena.

1.2 Conservation of Charge

One of the most fundamental principles in physics is the law of conservation of electric charge.

Law 1.2.1 (Conservation of Charge)**.** *The total electric charge in an isolated system remains constant. Charge can neither be created nor destroyed, only transferred from one body to another.*

Mathematically, for an isolated system:

$$Q_{\text{total}} = Q_1 + Q_2 + Q_3 + \cdots = \text{constant} \tag{1.1}$$

Key Point

In any physical process, the algebraic sum of all charges remains unchanged. If a charge $+q$ appears somewhere, an equal charge $-q$ must appear elsewhere.

1.2.1 Examples of Charge Conservation

Example

A neutral atom has equal numbers of protons and electrons. If an electron is removed (ionization), the atom becomes positively charged, but the removed electron carries away an equal negative charge. The total charge remains zero.
Initial state: $Q_{\text{total}} = Q_{\text{protons}} + Q_{\text{electrons}} = +ne + (-ne) = 0$
Final state: $Q_{\text{total}} = Q_{\text{ion}} + Q_{\text{free electron}} = +(n)e + (-(n-1)e - e) = 0$

1.3 Charge Quantization

Electric charge is quantized, meaning it exists only in discrete amounts, not as a continuous variable.

Law 1.3.1 (Charge Quantization)**.** *Any observable charge Q is an integer multiple of the elementary charge e:*

$$Q = ne \tag{1.2}$$

where n is an integer ($n = 0, \pm1, \pm2, \pm3, \ldots$) and $e = 1.602 \times 10^{-19}$ C

The elementary charge e is the magnitude of charge carried by a single proton (positive) or electron (negative).

Key Point

Charge cannot have arbitrary values. You cannot have $0.5e$ or $2.7e$ of charge; only integer multiples of e are possible.

Example

Calculate the number of electrons in 1 Coulomb of charge.
Solution:
Given: $Q = 1$ C, $e = 1.602 \times 10^{-19}$ C
Using $Q = ne$:

$$n = \frac{Q}{e} \tag{1.3}$$

$$= \frac{1}{1.602 \times 10^{-19}} \tag{1.4}$$

$$= 6.24 \times 10^{18} \text{ electrons} \tag{1.5}$$

This enormous number illustrates why charge seems continuous in everyday experience.

1.4 Coulomb's Law: The Electric Force

The force between two point charges is given by Coulomb's law, one of the fundamental laws of electromagnetism.

Law 1.4.1 (Coulomb's Law)**.** *The magnitude of the electric force between two point charges q_1 and q_2 separated by a distance r is:*

$$F = k\frac{|q_1 q_2|}{r^2} \tag{1.6}$$

where $k = 8.99 \times 10^9$ $N{\cdot}m^2/C^2$ is Coulomb's constant.

Alternatively, Coulomb's constant can be written as:

$$k = \frac{1}{4\pi\epsilon_0} \tag{1.7}$$

where $\epsilon_0 = 8.85 \times 10^{-12}$ $\mathrm{C}^2/(\mathrm{N}{\cdot}\mathrm{m}^2)$ is the permittivity of free space.

1.4.1 Vector Form of Coulomb's Law

The force is a vector quantity. The force on charge q_2 due to charge q_1 is:

$$\vec{F}_{12} = k\frac{q_1 q_2}{r^2}\hat{r}_{12} \tag{1.8}$$

where $\hat{r}_{12}$ is the unit vector pointing from q_1 to q_2.

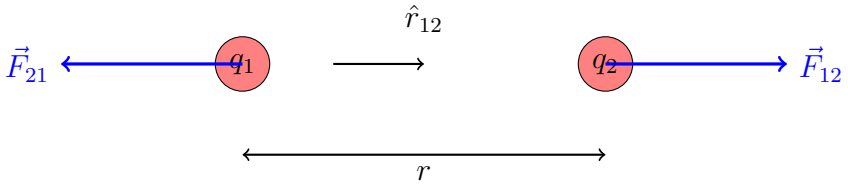


Figure 1.1: Electric force between two positive charges (repulsive)

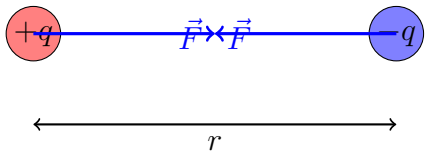


Figure 1.2: Electric force between opposite charges (attractive)

1.4.2 Properties of Electric Force

1. **Inverse Square Law**: Force decreases as $1/r^2$
2. **Central Force**: Acts along the line joining the charges
3. **Action-Reaction Pair**: $\vec{F}_{12} = -\vec{F}_{21}$ (Newton's third law)
4. **Superposition**: The total force on a charge due to multiple charges is the vector sum of individual forces

1.4.3 Superposition Principle

When multiple charges are present, the net force on any charge is the vector sum of forces from all other charges:

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots = \sum_{i=1}^{n} \vec{F}_i \tag{1.9}$$

Example

Three charges are arranged as shown: $q_1 = +2\mu$C at origin, $q_2 = -3\mu$C at (3,0) m, and $q_3 = +4\mu$C at (0,4) m. Find the net force on q_1.

Solution:

Force on q_1 due to q_2:

$$F_{12} = k\frac{|q_1 q_2|}{r_{12}^2} = 8.99 \times 10^9 \frac{(2 \times 10^{-6})(3 \times 10^{-6})}{3^2} \tag{1.10}$$

$$= 5.99 \times 10^{-3} \text{ N (attractive, toward } q_2) \tag{1.11}$$

$$\vec{F}_{12} = 5.99 \times 10^{-3}\hat{i} \text{ N} \tag{1.12}$$

Force on q_1 due to q_3:

$$F_{13} = k\frac{|q_1 q_3|}{r_{13}^2} = 8.99 \times 10^9 \frac{(2 \times 10^{-6})(4 \times 10^{-6})}{4^2} \tag{1.13}$$

$$= 4.49 \times 10^{-3} \text{ N (repulsive)} \tag{1.14}$$

$$\vec{F}_{13} = 4.49 \times 10^{-3}\hat{j} \text{ N} \tag{1.15}$$

Net force:

$$\vec{F}_{\text{net}} = \vec{F}_{12} + \vec{F}_{13} \tag{1.16}$$

$$= 5.99 \times 10^{-3}\hat{i} + 4.49 \times 10^{-3}\hat{j} \text{ N} \tag{1.17}$$

$$|\vec{F}_{\text{net}}| = \sqrt{(5.99)^2 + (4.49)^2} \times 10^{-3} = 7.49 \times 10^{-3} \text{ N} \tag{1.18}$$

1.5 Applications of Electric Force

1.5.1 Atomic Structure

The electric force holds electrons in orbit around the nucleus. For a hydrogen atom:

$$F = k\frac{e^2}{r^2} \approx 8.2 \times 10^{-8} \text{ N} \tag{1.19}$$

where $r \approx 0.53 \times 10^{-10}$ m (Bohr radius).

1.5.2 Electrostatic Precipitators

Used to remove particles from exhaust gases. Charged particles are attracted to oppositely charged plates by electric force.

1.5.3 Inkjet Printers

Charged ink droplets are deflected by electric forces to create patterns on paper.

Chapter 2

Electric Fields

2.1 The Concept of Electric Field

The electric field is a vector field that describes the electric force per unit charge at any point in space.

Important

The electric field $\vec{E}$ at a point is defined as the force per unit positive test charge:

$$\vec{E} = \frac{\vec{F}}{q_0} \tag{2.1}$$

where q_0 is a small positive test charge.

Units: Newton per Coulomb (N/C) or Volt per meter (V/m)

2.1.1 Physical Interpretation

The electric field represents the "influence" of a charge in the surrounding space. It exists independently of whether a test charge is present to experience it.

2.2 Electric Field Due to a Point Charge

For a point charge Q, the electric field at distance r is:

$$\vec{E} = k\frac{Q}{r^2}\hat{r} \tag{2.2}$$

Magnitude:

$$E = k\frac{|Q|}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{|Q|}{r^2} \tag{2.3}$$

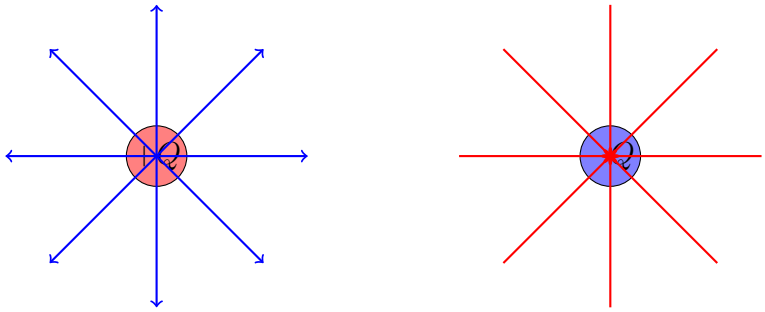
2.2.1 Derivation

Starting from Coulomb's law, the force on a test charge q_0 due to charge Q is:

$$\vec{F} = k\frac{Qq_0}{r^2}\hat{r} \tag{2.4}$$

By definition of electric field:

$$\vec{E} = \frac{\vec{F}}{q_0} = k\frac{Q}{r^2}\hat{r} \tag{2.5}$$



(a) Positive charge: field points outward (b) Negative charge: field points inward

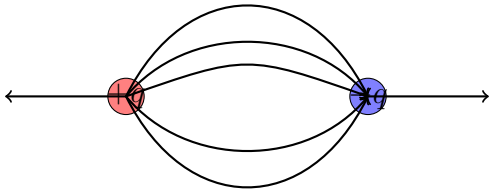
Figure 2.1: Electric field lines for point charges

2.3 Electric Field Lines (Lines of Force)

Electric field lines are imaginary lines that represent the direction and strength of the electric field.

2.3.1 Properties of Field Lines

1. Field lines begin on positive charges and end on negative charges
2. The tangent to a field line at any point gives the direction of $\vec{E}$
3. The density of lines indicates field strength
4. Field lines never cross
5. Field lines are perpendicular to conductor surfaces



Electric dipole field lines

Figure 2.2: Field lines for an electric dipole

2.4 Electric Field of Continuous Charge Distributions

For continuous charge distributions, we integrate the contributions from infinitesimal charge elements.

2.4.1 General Approach

$$\vec{E} = \int d\vec{E} = \int k\frac{dq}{r^2}\hat{r} \tag{2.6}$$

For different distributions:

- Linear: $dq = \lambda\, dl$ (charge per unit length)
- Surface: $dq = \sigma\, dA$ (charge per unit area)
- Volume: $dq = \rho\, dV$ (charge per unit volume)

2.5 Electric Field of a Ring of Charge

Consider a uniformly charged ring of radius R with total charge Q. Find the electric field at a point P on the axis at distance x from the center.

2.5.1 Derivation

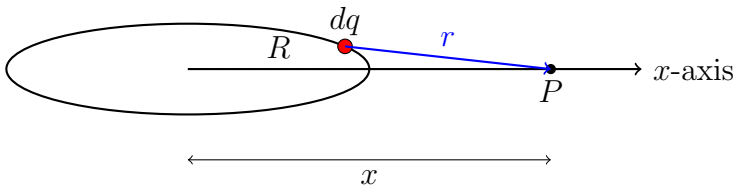


Figure 2.3: Ring of charge geometry

Consider an infinitesimal element dq on the ring. The distance from dq to point P is:

$$r = \sqrt{x^2 + R^2} \tag{2.7}$$

The electric field due to dq is:

$$dE = k\frac{dq}{r^2} = k\frac{dq}{x^2 + R^2} \tag{2.8}$$

By symmetry, the perpendicular components cancel, and only the axial component survives:

$$dE_x = dE\cos\theta = dE\frac{x}{r} = k\frac{dq}{x^2 + R^2}\cdot\frac{x}{\sqrt{x^2 + R^2}} \tag{2.9}$$

Simplifying:

$$dE_x = k\frac{x\, dq}{(x^2 + R^2)^{3/2}} \tag{2.10}$$

Integrating over the entire ring:

$$E_x = \int dE_x = \int k\frac{x\,dq}{(x^2+R^2)^{3/2}} \tag{2.11}$$

$$= \frac{kx}{(x^2+R^2)^{3/2}}\int dq \tag{2.12}$$

$$= \frac{kx}{(x^2+R^2)^{3/2}}\cdot Q \tag{2.13}$$

Important

Electric field on the axis of a uniformly charged ring:

$$\vec{E} = \frac{kQx}{(x^2+R^2)^{3/2}}\hat{i} \tag{2.14}$$

2.5.2 Special Cases

At the center $(x = 0)$:

$$E = 0 \tag{2.15}$$

This is expected by symmetry.

Far from the ring $(x \gg R)$:

$$E \approx \frac{kQ}{x^2} \tag{2.16}$$

The ring behaves like a point charge.

Example

A ring of radius 10 cm carries a uniform charge of 5μC. Find the electric field at a point 10 cm from the center along the axis.

Solution:

Given: $R = 0.1$ m, $Q = 5\times10^{-6}$ C, $x = 0.1$ m

$$E = \frac{kQx}{(x^2+R^2)^{3/2}} \tag{2.17}$$

$$= \frac{(8.99\times10^9)(5\times10^{-6})(0.1)}{[(0.1)^2+(0.1)^2]^{3/2}} \tag{2.18}$$

$$= \frac{4495}{(0.02)^{3/2}} \tag{2.19}$$

$$= \frac{4495}{0.00283} \tag{2.20}$$

$$= 1.59\times10^6 \text{ N/C} \tag{2.21}$$

2.6 Electric Field of a Disk of Charge

Consider a uniformly charged disk of radius R with surface charge density σ. Find the field at point P on the axis at distance x from the center.

2.6.1 Derivation

We can treat the disk as a collection of concentric rings. Consider a ring of radius r and thickness dr.

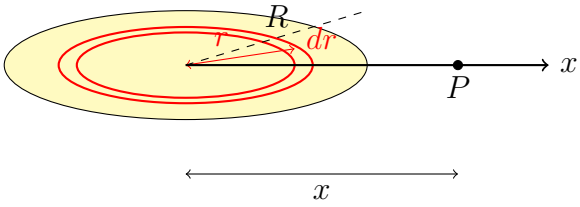


Figure 2.4: Disk of charge geometry

The area of the ring element is:

$$dA = 2\pi r\, dr \tag{2.22}$$

The charge on this ring is:

$$dq = \sigma\, dA = \sigma \cdot 2\pi r\, dr \tag{2.23}$$

Using the result for a ring of charge:

$$dE = \frac{k(dq)x}{(x^2 + r^2)^{3/2}} = \frac{k\sigma \cdot 2\pi r x\, dr}{(x^2 + r^2)^{3/2}} \tag{2.24}$$

Integrating from $r = 0$ to $r = R$:

$$E = \int_0^R \frac{2\pi k\sigma x r\, dr}{(x^2 + r^2)^{3/2}} \tag{2.25}$$

Let $u = x^2 + r^2$, then $du = 2r\, dr$:

$$E = 2\pi k\sigma x \int_{x^2}^{x^2+R^2} \frac{du/2}{u^{3/2}} \tag{2.26}$$

$$= \pi k\sigma x \left[-\frac{2}{u^{1/2}}\right]_{x^2}^{x^2+R^2} \tag{2.27}$$

$$= 2\pi k\sigma x \left[\frac{1}{x} - \frac{1}{\sqrt{x^2 + R^2}}\right] \tag{2.28}$$

$$= 2\pi k\sigma \left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right] \tag{2.29}$$

Using $k = 1/(4\pi\epsilon_0)$:

Important

Electric field on the axis of a uniformly charged disk:

$$E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right] \tag{2.30}$$

2.6.2 Infinite Plane of Charge

For an infinite plane ($R \to \infty$):

$$E = \frac{\sigma}{2\epsilon_0} \tag{2.31}$$

This remarkable result shows that the field is uniform and independent of distance from the plane!

2.7 Point Charge in an Electric Field

When a point charge q is placed in an external electric field $\vec{E}$, it experiences a force:

$$\vec{F} = q\vec{E} \tag{2.32}$$

If $q > 0$: force is in the direction of $\vec{E}$
If $q < 0$: force is opposite to $\vec{E}$

2.7.1 Motion in Uniform Electric Field

For a charge in a uniform field, the acceleration is:

$$\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m} \tag{2.33}$$

This is analogous to motion in a uniform gravitational field.

Example

An electron (mass $m = 9.11 \times 10^{-31}$ kg, charge $e = -1.6 \times 10^{-19}$ C) is placed in a uniform electric field of 1000 N/C pointing upward. Find its acceleration.
Solution:

$$F = |q|E = (1.6 \times 10^{-19})(1000) = 1.6 \times 10^{-16} \text{ N} \tag{2.34}$$

$$a = \frac{F}{m} = \frac{1.6 \times 10^{-16}}{9.11 \times 10^{-31}} \tag{2.35}$$

$$= 1.76 \times 10^{14} \text{ m/s}^2 \text{ (downward)} \tag{2.36}$$

The electron accelerates opposite to the field direction because it's negatively charged.

2.8 Electric Dipole in an Electric Field

An electric dipole consists of two equal and opposite charges $+q$ and $-q$ separated by distance d.

2.8.1 Dipole Moment

The electric dipole moment is defined as:

$$\vec{p} = q\vec{d} \tag{2.37}$$

where $\vec{d}$ points from the negative to the positive charge.

Unit: Coulomb-meter (C·m)

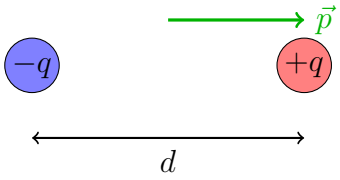


Figure 2.5: Electric dipole

2.8.2 Torque on a Dipole

When a dipole is placed in a uniform electric field $\vec{E}$, it experiences a torque:

Force on $+q$: $\vec{F}_+ = q\vec{E}$

Force on $-q$: $\vec{F}_- = -q\vec{E}$

Net force: $\vec{F}_{\text{net}} = 0$

However, these forces create a torque. If the dipole makes angle θ with the field:

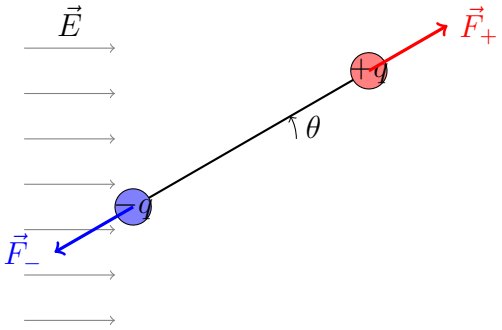


Figure 2.6: Dipole in uniform electric field

The magnitude of torque is:

$$\tau = F \cdot d \sin\theta = qEd \sin\theta \tag{2.38}$$

Since $p = qd$:

Important

Torque on an electric dipole:

$$\vec{\tau} = \vec{p} \times \vec{E} \tag{2.39}$$

Magnitude: $\tau = pE \sin\theta$

2.8.3 Potential Energy of a Dipole

The potential energy is:

$$U = -\vec{p} \cdot \vec{E} = -pE\cos\theta \tag{2.40}$$

- Minimum at $\theta = 0$ (aligned with field): $U = -pE$
- Maximum at $\theta = 180$ (opposite to field): $U = +pE$

Chapter 3

Gauss's Law

3.1 The Flux of a Vector Field

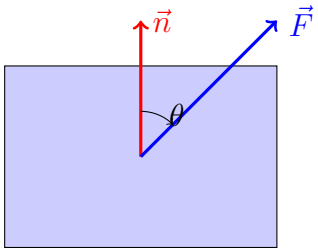
Before discussing electric flux, we need to understand the general concept of flux.

3.1.1 Definition of Flux

Flux represents the "flow" of a vector field through a surface. For a uniform field $\vec{F}$ through a flat surface of area A:

$$\Phi = \vec{F} \cdot \vec{A} = FA\cos\theta \tag{3.1}$$

where θ is the angle between $\vec{F}$ and the normal to the surface.



Area A

Figure 3.1: Flux through a surface

For a non-uniform field or curved surface:

$$\Phi = \int \vec{F} \cdot d\vec{A} \tag{3.2}$$

3.2 The Flux of Electric Field

Electric flux is the measure of the electric field passing through a surface.

Important

Electric flux through a surface:

$$\Phi_E = \int \vec{E} \cdot d\vec{A} \tag{3.3}$$

Unit: N·m^2/C or V·m

For a uniform field and flat surface:

$$\Phi_E = EA\cos\theta \tag{3.4}$$

3.2.1 Flux Through a Closed Surface

For a closed surface, we use:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} \tag{3.5}$$

The circle on the integral sign indicates integration over a closed surface.

3.3 Gauss's Law

Gauss's law is one of the four Maxwell equations and provides a powerful method for calculating electric fields.

Law 3.3.1 (Gauss's Law)**.** *The total electric flux through any closed surface equals the net charge enclosed divided by* ϵ_0*:*

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \tag{3.6}$$

3.3.1 Derivation for a Point Charge

Consider a point charge Q at the center of a spherical surface of radius r.

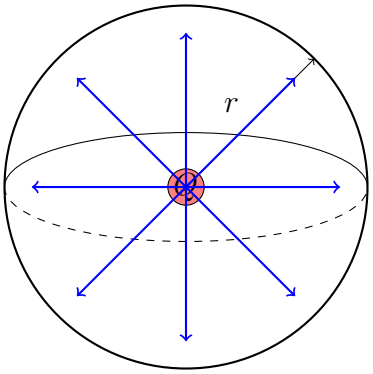


Figure 3.2: Point charge and spherical Gaussian surface

At every point on the sphere, $|\vec{E}| = kQ/r^2$ and $\vec{E}$ is perpendicular to the surface.

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E \, dA \tag{3.7}$$

$$= E \oint dA = E \cdot 4\pi r^2 \tag{3.8}$$

$$= \frac{kQ}{r^2} \cdot 4\pi r^2 \tag{3.9}$$

$$= 4\pi kQ = \frac{Q}{\epsilon_0} \tag{3.10}$$

This confirms Gauss's law for a point charge.

3.3.2 Key Points

Key Point

1. The Gaussian surface is imaginary
2. Only charges inside the surface contribute to flux
3. The field at the surface may be due to charges both inside and outside
4. Gauss's law is always true but useful only when symmetry exists

3.4 Applications of Gauss's Law

Gauss's law is most useful when the charge distribution has high symmetry:

- Spherical symmetry
- Cylindrical symmetry
- Planar symmetry

3.4.1 Strategy for Using Gauss's Law

1. Identify the symmetry
2. Choose an appropriate Gaussian surface
3. Calculate Q_{enc}
4. Evaluate $\oint \vec{E} \cdot d\vec{A}$
5. Solve for E

3.5 Spherically Symmetric Charge Distribution

3.5.1 Uniformly Charged Sphere

Consider a sphere of radius R with uniform volume charge density ρ.

Total charge: $Q = \rho \cdot \frac{4}{3}\pi R^3$

Case 1: Outside the sphere ($r > R$)

Choose a spherical Gaussian surface of radius $r > R$:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0} \tag{3.11}$$

$$E \cdot 4\pi r^2 = \frac{Q}{\epsilon_0} \tag{3.12}$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{kQ}{r^2} \tag{3.13}$$

The field outside is the same as if all charge were at the center.

Case 2: Inside the sphere ($r < R$)

The enclosed charge is:

$$Q_{\text{enc}} = \rho \cdot \frac{4}{3}\pi r^3 = Q\frac{r^3}{R^3} \tag{3.14}$$

Applying Gauss's law:

$$E \cdot 4\pi r^2 = \frac{Qr^3}{R^3\epsilon_0} \tag{3.15}$$

$$E = \frac{Qr}{4\pi\epsilon_0 R^3} = \frac{kQr}{R^3} \tag{3.16}$$

Important

Electric field of a uniformly charged sphere:

$$E = \begin{cases} \frac{kQr}{R^3} & r < R \\ \frac{kQ}{r^2} & r \geq R \end{cases} \tag{3.17}$$

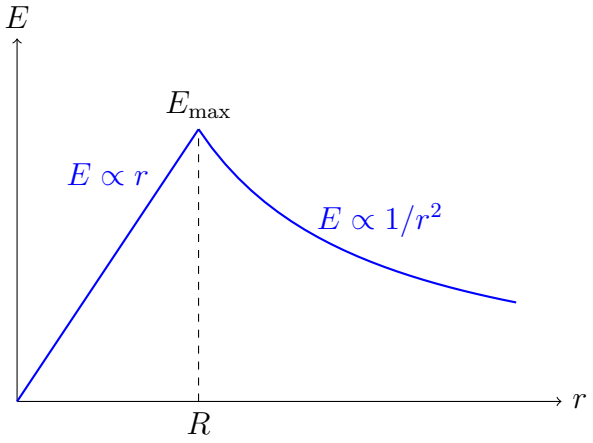


Figure 3.3: Electric field vs. distance for a uniformly charged sphere

3.6 Field of a Charged Isolated Conductor

Conductors have special properties:

1. Charges reside on the surface
2. Electric field inside is zero
3. Electric field at the surface is perpendicular to the surface

3.6.1 Field Inside a Conductor

For any Gaussian surface inside a conductor:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0} = 0 \tag{3.18}$$

Since $Q_{\text{enc}} = 0$ (no charge inside), and this is true for any internal Gaussian surface:

$$\vec{E} = 0 \quad \text{(inside conductor)} \tag{3.19}$$

3.6.2 Field at the Surface

Just outside the conductor surface with surface charge density σ:

Choose a cylindrical Gaussian surface partially inside and partially outside:

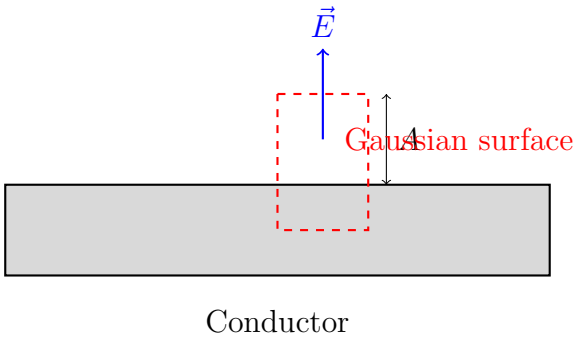


Figure 3.4: Gaussian surface at conductor surface

Only the outer cap contributes to flux:

$$\oint \vec{E} \cdot d\vec{A} = EA = \frac{Q_{\text{enc}}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0} \tag{3.20}$$

$$E = \frac{\sigma}{\epsilon_0} \tag{3.21}$$

Important

Electric field just outside a conductor:

$$E = \frac{\sigma}{\epsilon_0} \tag{3.22}$$

Chapter 4

Electric Potential and Potential Energy

4.1 Electric Potential Energy

The electric force is conservative, so we can define potential energy.

4.1.1 Work Done by Electric Force

For a point charge q moving in an electric field from point A to point B:

$$W_{AB} = \int_A^B \vec{F} \cdot d\vec{l} = q \int_A^B \vec{E} \cdot d\vec{l} \tag{4.1}$$

4.1.2 Definition of Potential Energy

The change in potential energy is:

$$\Delta U = U_B - U_A = -W_{AB} = -q \int_A^B \vec{E} \cdot d\vec{l} \tag{4.2}$$

4.1.3 Potential Energy of Two Point Charges

For two point charges q_1 and q_2 separated by distance r:

$$U = k\frac{q_1 q_2}{r} \tag{4.3}$$

Taking $U = 0$ at $r = \infty$.

4.2 Electric Potential

Electric potential is potential energy per unit charge.

Important

Electric potential at point P:

$$V_P = \frac{U_P}{q_0} \tag{4.4}$$

Unit: Volt (V) = Joule/Coulomb (J/C)

The potential difference between two points:

$$V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{l} \tag{4.5}$$

4.2.1 Physical Meaning

- Potential is a scalar quantity (easier to work with than vectors)
- Only potential *differences* are meaningful
- High potential = high potential energy for positive charges
- Positive charges tend to move from high to low potential

4.3 Calculating Potential from the Field

The potential difference is:

$$V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{l} \tag{4.6}$$

4.3.1 Uniform Electric Field

For a uniform field $\vec{E}$ in the x-direction:

$$V_B - V_A = -\int_A^B E\,dx = -E\int_A^B dx \tag{4.7}$$

$$= -E(x_B - x_A) = -Ed \tag{4.8}$$

where d is the displacement in the direction of the field.

Example

Two parallel plates separated by 2 cm have a potential difference of 100 V. Find the electric field between them.
Solution:

$$E = \frac{V}{d} = \frac{100}{0.02} = 5000 \text{ V/m} \tag{4.9}$$

The field points from the positive to the negative plate.

4.4 Potential Due to Point Charges

4.4.1 Single Point Charge

For a point charge Q, choosing $V = 0$ at infinity:

$$V(r) - V(\infty) = -\int_{\infty}^{r} \vec{E} \cdot d\vec{l} \tag{4.10}$$

$$= -\int_{\infty}^{r} \frac{kQ}{r^2}\, dr \tag{4.11}$$

$$= kQ \left[\frac{1}{r}\right]_{\infty}^{r} \tag{4.12}$$

$$= \frac{kQ}{r} \tag{4.13}$$

Important

Potential due to a point charge:

$$V = \frac{kQ}{r} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r} \tag{4.14}$$

4.4.2 Multiple Point Charges

By superposition:

$$V = \sum_{i=1}^{n} \frac{kq_i}{r_i} \tag{4.15}$$

Example

Find the potential at point P due to charges $q_1 = +2\mu$C at (0,0) and $q_2 = -3\mu$C at (3,0) m. Point P is at (0,4) m.

Solution:

Distances:

$$r_1 = 4 \text{ m} \tag{4.16}$$

$$r_2 = \sqrt{3^2 + 4^2} = 5 \text{ m} \tag{4.17}$$

Potential:

$$V = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} \tag{4.18}$$

$$= 8.99 \times 10^9 \left(\frac{2 \times 10^{-6}}{4} + \frac{-3 \times 10^{-6}}{5}\right) \tag{4.19}$$

$$= 8.99 \times 10^9 (0.5 - 0.6) \times 10^{-6} \tag{4.20}$$

$$= -899 \text{ V} \tag{4.21}$$

4.5 Potential Due to Continuous Charge Distributions

For a continuous distribution:

$$V = \int \frac{k\,dq}{r} \tag{4.22}$$

4.5.1 Uniformly Charged Ring

For a ring of radius R with total charge Q, the potential on the axis at distance x is:
All elements dq are at the same distance $r = \sqrt{x^2 + R^2}$ from the point:

$$V = \int \frac{k\,dq}{\sqrt{x^2 + R^2}} \tag{4.23}$$

$$= \frac{k}{\sqrt{x^2 + R^2}} \int dq \tag{4.24}$$

$$= \frac{kQ}{\sqrt{x^2 + R^2}} \tag{4.25}$$

Important

Potential on the axis of a charged ring:

$$V = \frac{kQ}{\sqrt{x^2 + R^2}} \tag{4.26}$$

Note: This is simpler than the electric field calculation (no vector components)!

4.5.2 Uniformly Charged Disk

For a disk of radius R with surface charge density σ:

$$V = \int_0^R \frac{k\sigma \cdot 2\pi r\,dr}{\sqrt{x^2 + r^2}} \tag{4.27}$$

$$= 2\pi k\sigma \int_0^R \frac{r\,dr}{\sqrt{x^2 + r^2}} \tag{4.28}$$

Let $u = x^2 + r^2$, $du = 2r\,dr$:

$$V = \pi k\sigma \left[2\sqrt{u}\right]_{x^2}^{x^2+R^2} \tag{4.29}$$

$$= 2\pi k\sigma \left(\sqrt{x^2 + R^2} - x\right) \tag{4.30}$$

$$= \frac{\sigma}{2\epsilon_0} \left(\sqrt{x^2 + R^2} - x\right) \tag{4.31}$$

4.6 Potential Due to an Electric Dipole

For a dipole with charges $+q$ and $-q$ separated by distance d:

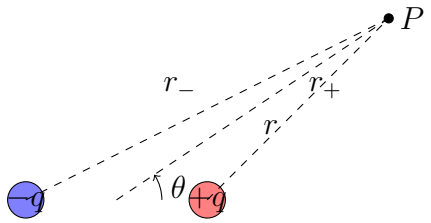


Figure 4.1: Electric dipole and field point

The potential is:

$$V = \frac{kq}{r_+} - \frac{kq}{r_-} = kq\left(\frac{1}{r_+} - \frac{1}{r_-}\right) \tag{4.32}$$

For $r \gg d$ (far field approximation):

$$r_+ \approx r - \frac{d}{2}\cos\theta, \quad r_- \approx r + \frac{d}{2}\cos\theta \tag{4.33}$$

$$V \approx kq\left(\frac{1}{r - \frac{d}{2}\cos\theta} - \frac{1}{r + \frac{d}{2}\cos\theta}\right) \tag{4.34}$$

$$\approx \frac{kqd\cos\theta}{r^2} \tag{4.35}$$

Since $p = qd$:

Important

Potential of a dipole (far field):

$$V = \frac{kp\cos\theta}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2} \tag{4.36}$$

4.7 Equipotential Surfaces

An equipotential surface is a surface on which the potential is constant.

4.7.1 Properties

1. Electric field lines are perpendicular to equipotential surfaces
2. No work is done moving a charge along an equipotential surface
3. Conductor surfaces are equipotential
4. Equipotentials are closer together where the field is stronger

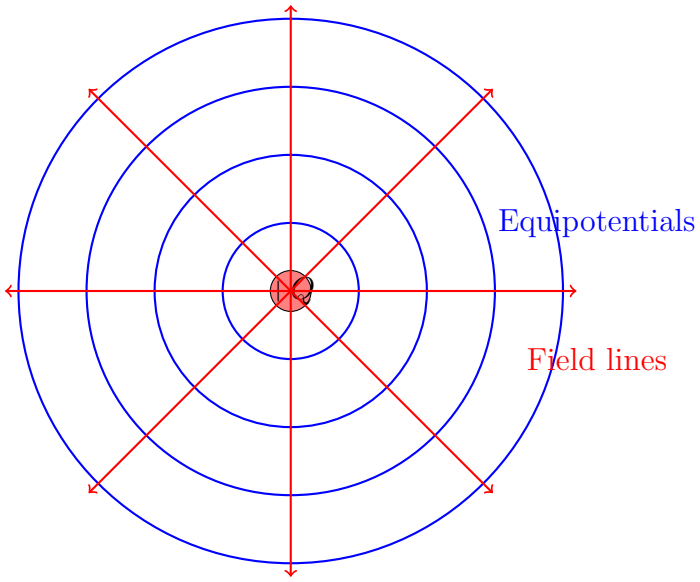


Figure 4.2: Equipotential surfaces and field lines for a point charge

4.8 Calculating Field from Potential

Since $V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{l}$, for an infinitesimal displacement:

$$dV = -\vec{E} \cdot d\vec{l} = -E_s \, ds \tag{4.37}$$

where E_s is the component of $\vec{E}$ in the direction of $d\vec{l}$.

4.8.1 Gradient

The electric field is the negative gradient of potential:

$$\vec{E} = -\nabla V = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right) \tag{4.38}$$

In component form:

$$E_x = -\frac{\partial V}{\partial x} \tag{4.39}$$

$$E_y = -\frac{\partial V}{\partial y} \tag{4.40}$$

$$E_z = -\frac{\partial V}{\partial z} \tag{4.41}$$

Example

The potential in a region is $V = 3x^2y - yz^2$ V. Find the electric field at point (2,1,3).

Solution:

$$E_x = -\frac{\partial V}{\partial x} = -6xy \tag{4.42}$$

$$E_y = -\frac{\partial V}{\partial y} = -(3x^2 - z^2) \tag{4.43}$$

$$E_z = -\frac{\partial V}{\partial z} = 2yz \tag{4.44}$$

At (2,1,3):

$$E_x = -6(2)(1) = -12 \text{ V/m} \tag{4.45}$$

$$E_y = -(3(4) - 9) = -3 \text{ V/m} \tag{4.46}$$

$$E_z = 2(1)(3) = 6 \text{ V/m} \tag{4.47}$$

$\vec{E} = -12\hat{i} - 3\hat{j} + 6\hat{k}$ V/m

Chapter 5

Electric Current and Resistance

5.1 Electric Current

Electric current is the rate of flow of electric charge.

Important

Electric current:

$$I = \frac{dQ}{dt} \tag{5.1}$$

Unit: Ampere (A) = Coulomb/second (C/s)

5.1.1 Direction of Current

By convention, current flows in the direction of positive charge motion (opposite to electron flow).

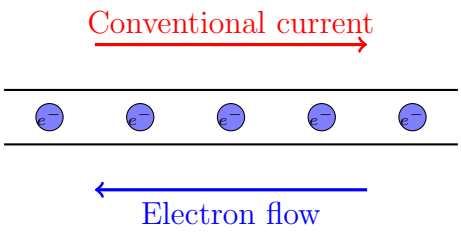


Figure 5.1: Current direction vs. electron flow

5.1.2 Microscopic View

If n is the number density of charge carriers, each with charge q, moving with drift velocity v_d through a wire of cross-sectional area A:

Number of charges passing through in time dt:

$$dN = n \cdot A \cdot v_d \, dt \tag{5.2}$$

Total charge:

$$dQ = q \, dN = nqAv_d \, dt \tag{5.3}$$

Current:

$$I = \frac{dQ}{dt} = nqAv_d \tag{5.4}$$

5.2 Current Density

Current density is current per unit area.

$$\vec{J} = \frac{I}{A}\hat{n} = nq\vec{v}_d \tag{5.5}$$

Unit: $\mathrm{A/m^2}$
For non-uniform current:

$$I = \int \vec{J} \cdot d\vec{A} \tag{5.6}$$

5.3 Resistance and Resistivity

5.3.1 Resistance

Resistance is the opposition to current flow.

Important

Resistance:

$$R = \frac{V}{I} \tag{5.7}$$

Unit: Ohm (Ω) = Volt/Ampere (V/A)

5.3.2 Resistivity

Resistivity ρ is a material property. For a conductor of length L and cross-sectional area A:

$$R = \rho\frac{L}{A} \tag{5.8}$$

Unit: Ohm-meter (Ω·m)

5.3.3 Conductivity

Conductivity is the inverse of resistivity:

$$\sigma = \frac{1}{\rho} \tag{5.9}$$

Unit: Siemens per meter (S/m) or $(\Omega \cdot \mathrm{m})^{-1}$
Relating current density to electric field:

$$\vec{J} = \sigma\vec{E} \tag{5.10}$$

Example

A copper wire has length 2 m, diameter 1 mm, and resistivity 1.7×10^{-8} Ω·m. Find its resistance.
Solution:

$$A = \pi r^2 = \pi(0.5 \times 10^{-3})^2 = 7.85 \times 10^{-7} \text{ m}^2 \tag{5.11}$$

$$R = \rho \frac{L}{A} = 1.7 \times 10^{-8} \times \frac{2}{7.85 \times 10^{-7}} \tag{5.12}$$

$$= 0.043\,\Omega = 43 \text{ m}\Omega \tag{5.13}$$

5.4 Ohm's Law

Law 5.4.1 (Ohm's Law)**.** *For many materials (ohmic conductors), the voltage across the conductor is proportional to the current through it:*

$$V = IR \tag{5.14}$$

Key Point

Ohm's law is not a fundamental law of physics but an empirical relationship that holds for many materials under certain conditions.

5.4.1 Microscopic Form

$$\vec{E} = \rho \vec{J} \tag{5.15}$$

5.4.2 Power Dissipation

Power dissipated in a resistor:

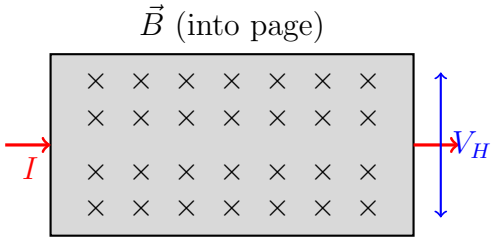
$$P = IV = I^2R = \frac{V^2}{R} \tag{5.16}$$

Example

A 100 Ω resistor carries a current of 0.5 A. Find (a) the voltage across it, and (b) the power dissipated.
Solution:
(a) $V = IR = (0.5)(100) = 50$ V
(b) $P = I^2R = (0.5)^2(100) = 25$ W
Alternatively: $P = IV = (0.5)(50) = 25$ W

5.5 The Hall Effect

The Hall effect is the production of a voltage difference across a conductor when a magnetic field is applied perpendicular to the current flow.



Hall effect in a conductor

Figure 5.2: Hall effect setup

When current I flows through a conductor of thickness t in a magnetic field B, a Hall voltage appears:

$$V_H = \frac{IB}{nqt} \tag{5.17}$$

where n is the charge carrier density and q is the charge.

5.5.1 Applications

- Determining charge carrier density
- Determining sign of charge carriers
- Magnetic field sensors

Chapter 6

Magnetic Forces and Fields

6.1 Magnetic Force on a Current

A current-carrying wire in a magnetic field experiences a force.

Important

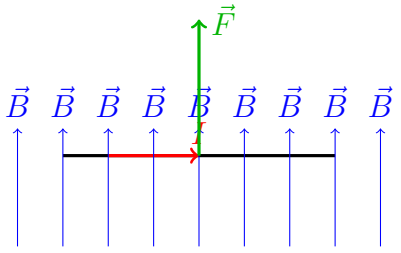
Magnetic force on a current element:

$$d\vec{F} = Id\vec{l} \times \vec{B} \tag{6.1}$$

For a straight wire of length L:

$$\vec{F} = I\vec{L} \times \vec{B} \tag{6.2}$$

Magnitude: $F = ILB\sin\theta$



Current perpendicular to magnetic field

Figure 6.1: Magnetic force on a current-carrying wire

6.1.1 Direction: Right-Hand Rule

Point fingers in direction of current, curl them toward $\vec{B}$, thumb points in direction of $\vec{F}$.

Example

A wire carrying 5 A of current in the $+x$ direction is placed in a magnetic field of 0.3 T in the $+y$ direction. Find the force per unit length.
Solution:

$$\vec{F} = I\vec{L} \times \vec{B} \tag{6.3}$$

$$\frac{\vec{F}}{L} = I(\hat{i}) \times (B\hat{j}) = IB(\hat{i} \times \hat{j}) \tag{6.4}$$

$$= (5)(0.3)\hat{k} = 1.5\hat{k} \text{ N/m} \tag{6.5}$$

The force is in the $+z$ direction.

6.2 The Biot-Savart Law

The Biot-Savart law gives the magnetic field produced by a current element.

Law 6.2.1 (Biot-Savart Law)**.** *The magnetic field at point P due to a current element $Id\vec{l}$ is:*

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \times \hat{r}}{r^2} \tag{6.6}$$

where $\mu_0 = 4\pi \times 10^{-7}$ $T{\cdot}m/A$ is the permeability of free space.

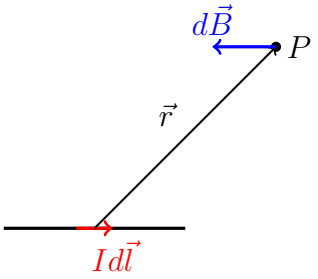


Figure 6.2: Biot-Savart law geometry

6.2.1 Magnetic Field of a Straight Wire

For an infinite straight wire carrying current I, the field at perpendicular distance r is:

$$B = \frac{\mu_0 I}{2\pi r} \tag{6.7}$$

The field lines form concentric circles around the wire.

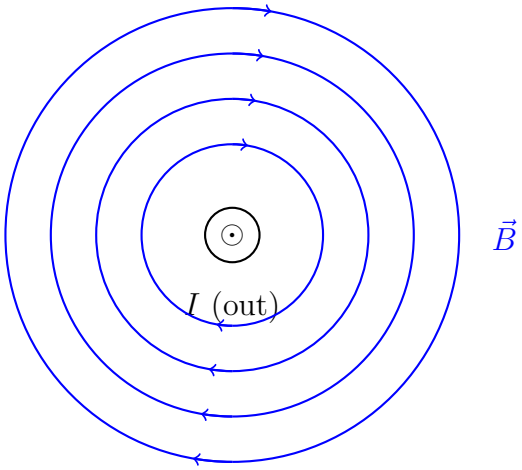


Figure 6.3: Magnetic field around a current-carrying wire

6.2.2 Derivation for Straight Wire

Consider an infinite wire along the z-axis carrying current I. Find B at distance r from the wire.

For an element dz at position z:

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{dz\hat{k} \times \vec{r}}{|\vec{r}|^3} \tag{6.8}$$

where $\vec{r}$ points from the element to the field point.

By symmetry, only the perpendicular component survives:

$$dB = \frac{\mu_0 I}{4\pi} \frac{dz \sin\alpha}{r^2 + z^2} \tag{6.9}$$

where $\sin\alpha = r/\sqrt{r^2 + z^2}$:

$$dB = \frac{\mu_0 I r\, dz}{4\pi(r^2 + z^2)^{3/2}} \tag{6.10}$$

Integrating from $-\infty$ to $+\infty$:

$$B = \frac{\mu_0 I r}{4\pi} \int_{-\infty}^{+\infty} \frac{dz}{(r^2 + z^2)^{3/2}} \tag{6.11}$$

$$= \frac{\mu_0 I r}{4\pi} \left[\frac{z}{r^2\sqrt{r^2 + z^2}} \right]_{-\infty}^{+\infty} \tag{6.12}$$

$$= \frac{\mu_0 I}{2\pi r} \tag{6.13}$$

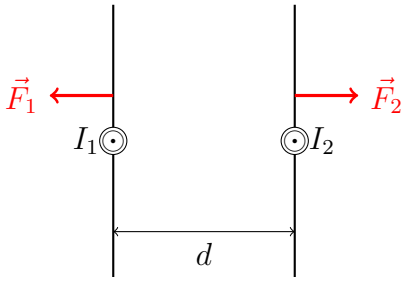
6.3 Magnetic Field Lines

Properties similar to electric field lines, but:

- No magnetic monopoles (no beginning or end)
- Field lines form closed loops

6.4 Force Between Two Parallel Conductors

Two parallel wires carrying currents exert forces on each other.



Parallel currents: repulsive? No, attractive!

Figure 6.4: Force between parallel currents

Wire 1 creates field $B_1 = \mu_0 I_1/(2\pi d)$ at wire 2.
Force per unit length on wire 2:

$$\frac{F}{L} = I_2 B_1 = \frac{\mu_0 I_1 I_2}{2\pi d} \tag{6.14}$$

Important

Force per unit length between parallel conductors:

$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d} \tag{6.15}$$

- Parallel currents: attractive
- Antiparallel currents: repulsive

6.5 Ampère's Law

Ampère's law relates the magnetic field around a closed loop to the current passing through it.

Law 6.5.1 (Ampère's Law)**.**

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} \tag{6.16}$$

Similar to Gauss's law for electric fields, Ampère's law is useful when high symmetry exists.

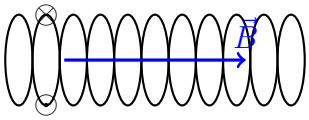
6.5.1 Application: Field of a Solenoid

A solenoid is a long coil of wire. For an ideal solenoid with n turns per unit length:
Inside the solenoid:

$$B = \mu_0 n I \tag{6.17}$$

Outside the solenoid:

$$B \approx 0 \tag{6.18}$$



Magnetic field inside a solenoid

Figure 6.5: Solenoid and its magnetic field

6.5.2 Derivation

Consider a rectangular Amperian loop with one side inside and one outside the solenoid:

$$\oint \vec{B} \cdot d\vec{l} = B \cdot L + 0 + 0 + 0 = BL \tag{6.19}$$

$$I_{\text{enc}} = nLI \tag{6.20}$$

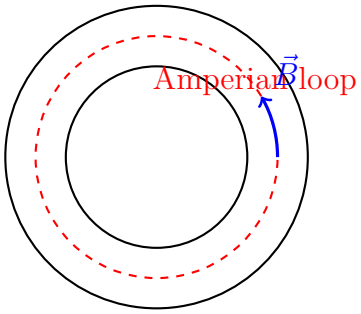
By Ampère's law:

$$BL = \mu_0 nLI \tag{6.21}$$

$$B = \mu_0 nI \tag{6.22}$$

6.6 Toroid

A toroid is a solenoid bent into a circle.



Cross-section of toroid

Figure 6.6: Toroid geometry

For a toroid with N total turns and radius r (measured to the center of the coil):

$$B = \frac{\mu_0 NI}{2\pi r} \tag{6.23}$$

The field is confined to the interior and decreases with radius.

Chapter 7

Electromagnetic Induction

7.1 Faraday's Experiments

Michael Faraday discovered that a changing magnetic field induces an electric current.

Key observations:

1. Moving a magnet near a coil induces current
2. Changing current in one coil induces current in a nearby coil
3. No current when magnetic field is constant

7.2 Faraday's Law of Induction

Law 7.2.1 (Faraday's Law)**.** *The induced emf in a closed loop equals the negative rate of change of magnetic flux through the loop:*

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \tag{7.1}$$

The magnetic flux is:

$$\Phi_B = \int \vec{B} \cdot d\vec{A} \tag{7.2}$$

For a uniform field and flat surface:

$$\Phi_B = BA\cos\theta \tag{7.3}$$

Unit of flux: Weber (Wb) = T·m^2

7.2.1 For N Turns

If a coil has N turns:

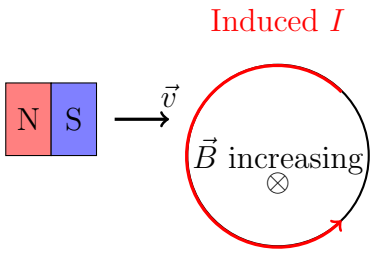
$$\mathcal{E} = -N\frac{d\Phi_B}{dt} \tag{7.4}$$

7.3 Lenz's Law

The negative sign in Faraday's law embodies Lenz's law.

Law 7.3.1 (Lenz's Law)**.** *The direction of induced current is such that it opposes the change in magnetic flux that produced it.*

This is a consequence of energy conservation.



Magnet approaching: flux into page increases
Induced current creates field out of page (opposes increase)

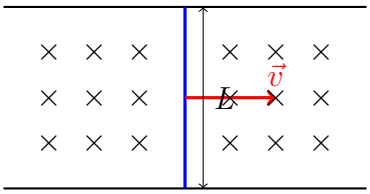
Figure 7.1: Lenz's law illustration

7.4 Motional EMF

When a conductor moves through a magnetic field, charges experience a force, creating an emf.

7.4.1 Derivation

Consider a rod of length L moving with velocity $\vec{v}$ perpendicular to magnetic field $\vec{B}$:



$\vec{B}$ into page

Figure 7.2: Motional emf setup

Charges in the rod experience Lorentz force:

$$\vec{F} = q\vec{v} \times \vec{B} \tag{7.5}$$

This separates charges, creating an electric field and potential difference:

$$\mathcal{E} = BLv \tag{7.6}$$

Important

Motional emf:

$$\mathcal{E} = BLv \tag{7.7}$$

for a rod of length L moving perpendicular to field B with velocity v.

7.4.2 Alternative Derivation Using Faraday's Law

In time dt, the rod moves distance $dx = v\,dt$.

Change in flux:

$$d\Phi_B = B \cdot L \cdot dx = BLv\,dt \tag{7.8}$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -BLv \cdot \frac{dt}{dt} = BLv \tag{7.9}$$

(The negative sign indicates direction by Lenz's law)

Example

A 20 cm rod moves at 5 m/s perpendicular to a 0.8 T magnetic field. Find the induced emf.

Solution:

$$\mathcal{E} = BLv \tag{7.10}$$

$$= (0.8)(0.2)(5) \tag{7.11}$$

$$= 0.8 \text{ V} \tag{7.12}$$

7.5 Induced Electric Field

A changing magnetic flux creates an induced electric field, even in the absence of conductors.

7.5.1 Induced Electric Field from Faraday's Law

From Faraday's law:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \tag{7.13}$$

This is fundamentally different from electrostatic fields:

- Electrostatic field: $\oint \vec{E} \cdot d\vec{l} = 0$ (conservative)
- Induced field: $\oint \vec{E} \cdot d\vec{l} \neq 0$ (non-conservative)

Example

A uniform magnetic field B perpendicular to a circular region of radius R increases at rate dB/dt. Find the induced electric field at distance r from the center.
Solution:
For $r < R$:
Flux through circle of radius r:

$$\Phi_B = B\pi r^2 \tag{7.14}$$

By Faraday's law:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \tag{7.15}$$

$$E(2\pi r) = -\pi r^2 \frac{dB}{dt} \tag{7.16}$$

$$E = -\frac{r}{2}\frac{dB}{dt} \tag{7.17}$$

The field is tangent to circles centered on the axis.

7.6 Maxwell's Equations: The Basic Equations of Electromagnetism

We can now summarize the fundamental laws of electromagnetism.

Important

Maxwell's Equations (Integral Form):

$$\text{Gauss's Law:} \quad \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0} \tag{7.18}$$

$$\text{No magnetic monopoles:} \quad \oint \vec{B} \cdot d\vec{A} = 0 \tag{7.19}$$

$$\text{Faraday's Law:} \quad \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \tag{7.20}$$

$$\text{Ampère-Maxwell Law:} \quad \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}} + \mu_0\epsilon_0\frac{d\Phi_E}{dt} \tag{7.21}$$

7.7 The Displacement Current

Maxwell added a term to Ampère's law to make it consistent.

7.7.1 The Problem

Ampère's law as originally stated:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}} \tag{7.22}$$

This fails for time-varying situations, such as a capacitor being charged.

7.7.2 Maxwell's Correction

Maxwell introduced the displacement current:

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt} \tag{7.23}$$

The complete Ampère-Maxwell law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0(I + I_d) = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \tag{7.24}$$

7.7.3 Physical Meaning

The displacement current is not a real current (no moving charges), but it has the same magnetic effect as a real current. It represents the time-varying electric field.

Example

A parallel-plate capacitor with circular plates of radius $R = 5$ cm is being charged. The electric field between plates increases at 10^{12} V/(m·s). Find the displacement current.

Solution:

Electric flux through the plates:

$$\Phi_E = EA = E\pi R^2 \tag{7.25}$$

Displacement current:

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 \pi R^2 \frac{dE}{dt} \tag{7.26}$$

$$= (8.85 \times 10^{-12})(\pi)(0.05)^2(10^{12}) \tag{7.27}$$

$$= 0.0696 \text{ A} \approx 70 \text{ mA} \tag{7.28}$$

7.8 Induced Magnetic Field

Just as a changing magnetic field induces an electric field (Faraday's law), a changing electric field induces a magnetic field (Ampère-Maxwell law).

For a changing electric field with cylindrical symmetry:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \tag{7.29}$$

7.8.1 Symmetry Between E and B

Changing B creates E	**Changing E creates B**
$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$	$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$

This symmetry led Maxwell to predict electromagnetic waves traveling at speed:

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m/s} \tag{7.30}$$

This is the speed of light!

Summary

Key Concepts

1. **Electric Charge**: Fundamental property; conserved and quantized
2. **Electric Force**: $\vec{F} = k\frac{q_1 q_2}{r^2}\hat{r}$ (Coulomb's law)
3. **Electric Field**: $\vec{E} = \vec{F}/q_0$; created by charges
4. **Gauss's Law**: $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$
5. **Electric Potential**: $V = U/q_0$; scalar quantity
6. **Current**: $I = dQ/dt$; flow of charge
7. **Resistance**: $R = V/I$ (Ohm's law)
8. **Magnetic Force**: $\vec{F} = I\vec{L} \times \vec{B}$
9. **Biot-Savart Law**: Gives $\vec{B}$ from current
10. **Ampère's Law**: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$
11. **Faraday's Law**: $\mathcal{E} = -d\Phi_B/dt$
12. **Maxwell's Equations**: Complete description of electromagnetism

Important Constants

- Elementary charge: $e = 1.602 \times 10^{-19}$ C
- Coulomb constant: $k = 8.99 \times 10^9$ N·m^2/C^2
- Permittivity: $\epsilon_0 = 8.85 \times 10^{-12}$ C^2/(N·m^2)
- Permeability: $\mu_0 = 4\pi \times 10^{-7}$ T·m/A
- Speed of light: $c = 3 \times 10^8$ m/s

Appendix A

Problem Set

A.1 Electric Force and Field

1. Two charges $q_1 = 3\mu\text{C}$ and $q_2 = -2\mu\text{C}$ are separated by 0.5 m. Find the force between them.

2. Calculate the electric field at the center of a square with 10 nC charges at each corner. Side length = 10 cm.

3. A ring of radius 5 cm has charge 20 nC. Find the field at 12 cm along the axis.

A.2 Gauss's Law

1. A spherical shell of radius 10 cm carries charge 50 nC. Find the field at (a) $r = 5$ cm, (b) $r = 15$ cm.

2. Use Gauss's law to find the field of an infinite line charge with linear density $\lambda = 5$ nC/m.

A.3 Electric Potential

1. Find the potential at distance 30 cm from a 5 μC charge.

2. Two charges $+q$ and $-q$ are at $(\pm a, 0)$. Find the potential at $(0, y)$.

A.4 Current and Resistance

1. A wire carries 2 A. How many electrons pass a point in 10 s?

2. A 50 Ω resistor carries 3 A. Find (a) voltage, (b) power dissipated.

A.5 Magnetism

1. A wire carries 10 A in a 0.5 T field. Find force per unit length.

2. Find the field at the center of a circular loop of radius 10 cm carrying 5 A.

A.6 Induction

1. A coil of 100 turns has area 0.1 m^2. Magnetic field changes from 0.2 T to 0.8 T in 0.5 s. Find induced emf.

2. A 30 cm rod moves at 4 m/s perpendicular to 0.6 T field. Find induced emf.

End of Textbook